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maximum entropy spectral analysis

Investigation of Level Changes of Lake Titicaca by Maximum Entropy Spectral Analysis

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With 2 Figures

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Summary

The time series of the yearly rise (maximum minus previous minimum) of the Titicaca lake level (16°S , Peru-Bolivia) for the years 1915 to 1981 is investigated by maximum entropy spectral analysis. The relative spectral density clearly shows peaks with periods of 10.6 and 2.4 years. An additional peak with a period of 4.7 years is extremely weak. The peak at 10.6 years is assigned to the fluctuations of the sunspot numbers and the peak at 2.4 years to the stratospheric quasi-biennial wind and temperature oscillation of the tropics. The most recent level fluctuations are reported in Table 1.

Zusammenfassung

Spektralanalytische Untersuchung der Pegeländerung des Titicacasees mit der Maximum-Entropie-Methode

Es wird die Zeitreihe des jährlichen Pegelanstiegs (Maximum minus vorhergehendes Minimum) des Titicacasees (16°S , Peru-Bolivien) für 1915 bis 1981 spektralanalytisch mit der Maximum-Entropie-Methode untersucht. Die relative Spektraldichte zeigt deutlich Spitzen bei Perioden um 10.6 und 2.4 Jahre. Eine weitere Spitze bei einer Periode um 4.7 Jahre ist nur extrem schwach angedeutet. Die Spitze um 10.6 Jahre wird der Schwankung der Sonnenfleckenzahlen zugeordnet, die Spitze um 2.4 Jahre der quasi-zweijährigen Schwankung der tropischen Höhenströmung und Temperatur. Tabelle 1 enthält Informationen über die jüngsten monatlichen Pegeländerungen.

* Dedicated to Prof. H. E. Landsberg on the occasion of his 80th birthday, in memoriam Prof. Landsberg (1906–1985).

1. Introduction

In previous investigations about the water balance of Lake Titicaca [19, 20, 22] it was shown that the yearly and interannual variations are mainly influenced by the precipitation regime. With the analysis of the level fluctuations consequently one also gets an insight into the variations of the general circulation of the atmosphere in this region. In that manner Kessler [20, 21] was already able to demonstrate a connection between upper air data and lake levels. In the summer rainy season the Altiplano lies within the region of the tropical easterlies. The winter drought is determined by the position and intensity of the subtropical westerly jet stream.

2. Data

In a previous spectral analysis [11] the monthly gauge readings of the lake level at the end of each month for the years 1914 to 1974 were used. The result had remained unsatisfactory since the spectrum did not even show the easily visible cycles of the time series with periods greater than 1 and less than 20 years. By theoretical reasoning and by carefully checking the data Künzel [23] has demonstrated that the sampling theorem had been partially disregarded.

In the present study the time series of the yearly rise of the lake level is analysed (see Fig. 1). In general the rise occurs in October to April. In the period 1915 to 1981 the mean yearly rise of the lake level amounts to 75 cm. The following data sources were used for 1914 to 1954: World Weather Records 1941–1950, p. 1335, Washington 1959; for 1955 to 1973: Arch. Met. Geoph. Biokl., Ser. B 24, p. 204, 1976; for 1974 to 1981: see Table 1.

Fig. 1. Time series of the yearly rise of the Titicaca lake level (maximum minus previous minimum, generally October to April)

Table 1. *Height of the Titicaca Lake Level (Puno) at the End of Each Month (1974–1981).*Units: 3799.0 + y meter

	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sep	Oct	Nov	Dec
1974	3.33	4.09	4.27	4.29	4.16	4.03	3.92	3.91	3.85	3.77	3.66	3.67
1975	3.87	4.39	4.62	4.60	4.54	4.43	4.31	4.19	4.11	4.05	3.95	4.08
1976	4.52	4.74	4.97	4.89	4.81	4.70	4.58	4.46	4.43	4.30	4.17	4.13
1977	4.12	4.36	4.67	4.62	4.52	4.39	4.29	4.15	4.09	4.01	4.02	4.06
1978	4.44	4.75	4.91	4.90	4.76	4.64	4.51	4.39	4.29	4.15	4.19	4.31
1979	4.78	4.96	5.16	5.16	5.07	4.91	4.78	4.65	4.52	4.47	4.36	4.40
1980	4.49	4.52	4.78	4.73	4.59	4.46	4.34	4.26	4.22	4.18	4.08	4.03
1981	4.25	4.58	4.89	4.96	4.85	4.71	4.57					

3. Maximum Entropy Spectral Estimation

Be X_t a discrete-time, wide-sense stationary, ergodic-mean, ergodic-covariance, real stochastic process and Δt the constant time interval. Moreover be $\underline{X}' = (X_1, \dots, X_{m+1})$ the random variables of X_t and $\underline{x}' = (x_1, \dots, x_{m+1})$ their realizations, ' denoting transposed. The entropy of $\underline{X}'$ is defined as

$$H\{\underline{X}'\} = - \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} f(\underline{x}') \cdot \ln(c^{m+1} \cdot f(\underline{x}')) \cdot dx_1 \cdot \dots \cdot dx_{m+1}, \quad (1)$$

where $f(\underline{x}')$ is the joint probability density function of $\underline{X}'$. Be $\underline{X} \sim N(\underline{\mu}, \underline{\Sigma})$ where $E\{\underline{X}\} = \underline{\mu} := (\mu, \dots, \mu)'$, $\underline{1} := (1, \dots, 1)'$, $\underline{\Sigma} = E\{(\underline{X} - \underline{\mu}\underline{1})'(\underline{X} - \underline{\mu}\underline{1})\}$; $E\{\cdot\}$ denotes the expectation operator and $\underline{\Sigma}$ the $(m+1) \times (m+1)$ Toeplitz covariance matrix with elements $\phi_0, \dots, \phi_m$. With the above assumptions the entropy of $\underline{X}'$ yields

$$H\{\underline{X}'\} = 1/2 \cdot \ln(\det\{\underline{\Sigma}\}), \quad (2)$$

where in (1) the constant c is set to $(2 \cdot \pi \cdot e)^{1/2}$ [30] and $\det\{\cdot\}$ denotes determinant. The spectral density $S(f)$ is determined by the entropy rate of (2)

$$h = \lim_{m \rightarrow \infty} \frac{H\{\underline{X}'\}}{m+1} = 1/2 \cdot \ln(2 \cdot f_N) + 1/(4 \cdot f_N) \cdot \int_{-f_N}^{+f_N} \ln(S(f)) \cdot df. \quad (3)$$

f_N being the Nyquist frequency $1/(2 \cdot \Delta t)$. Details may be found in [16, 33]. Suppose one has available $n \geq m$ values generated by X_t of order m . Unlike the classical approach [8] which assumes arbitrary $\phi_k = 0$ for $|k| \geq m$

Burg [9] proposed an extension which makes the entropy rate (3) stationary with respect to the unknown ϕ_k 's for $|k| \geq m$. The final result is

$$\hat{S}(f) = \frac{\Delta t \cdot \hat{P}_m}{\left| 1 + \sum_{k=1}^m \hat{\gamma}_{mk} \cdot e^{-i \cdot 2 \cdot \pi \cdot f \cdot k \cdot \Delta t} \right|^2} - f_N \leq f \leq f_N \quad (4)$$

where $\hat{\cdot}$ denotes an estimate. The spectral density estimation has been reduced to the estimation of a prediction error filter with coefficients $1, \hat{\gamma}_{m1}, \dots, \hat{\gamma}_{mm}$ and prediction error power $\hat{P}_m$. For further mathematical details see [15, 33].

The investigation by Künzel [23] utilizes the Burg algorithm [10] for the estimation of the parameters of (4). The algorithm is described in detail by Andersen [3, 4]. Künzel [23] has implemented a slight modification according to [13] which in cases of long prediction error filters saves computer storage. To make the filter order selection problem more objective three selection criteria have been implemented: The final prediction error criterion (*FPE*) [1, 2], the criterion autoregressive transfer function (*CAT*) [29] and the optimum Bayes decision rule (*OBD*) [32]. The *OBD* is the only consistent criterion used thus limiting its usefulness in small sample cases.

$$FPE_m = (n + m + 1)/(n - m - 1) \cdot \hat{P}_m \quad (m = 0, \dots, mmax) \quad (5)$$

$$CAT_m = 1/n \cdot \sum_{k=1}^m PW_k^{-1} - PW_m^{-1}$$

$$PW_k = n/(n - k) \cdot \hat{P}_k \quad (m = 1, \dots, mmax) \quad (6)$$

$$OBD_m = (n - m - 2) \cdot \ln(\hat{P}_m) + m \cdot \ln(n)$$

$$+ \sum_{k=0}^{m-1} \ln(\hat{P}_k) + \sum_{k=1}^m \hat{\gamma}_{mk}^2 \quad (m = 0, \dots, mmax) \quad (7)$$

The filter order m which minimizes the above criteria points to the correct order. In general *FPE* and *CAT* lead to reasonable results in small sample cases [6]. Nevertheless is any filter order selection uncertain.

It is well known that the position of the spectral peaks is somewhat dependent on the initial phase when using the Burg algorithm, the phase dependency being most pronounced in the case of truncated sinusoids with high signal to noise ratio (*SNR*). Here we define the *SNR* as

$$SNR = \hat{\phi}_0 / \hat{P}_m = \hat{P}_0 / \hat{P}_m \quad (8)$$

Amongst others Swingler [35, 36, 37] made great efforts in analysing and overcoming this problem. Based on his work Kaveh and Lippert [18] developed a modified Burg algorithm which they called the optimum tapered Burg algorithm. This algorithm minimizes the power of the frequency errors over all possible phases, but does not affect the high resolution property of the Burg algorithm. Their paper should be read carefully since some formulas have to be corrected for typing errors. The corrected formulas together with a subroutine written in FORTRAN 77 may be found in [23].

The spectral peaks of (4) have been determined by a systematic search procedure proposed by Radoski et al. [31].

4. Results

The relative spectral density of the time series of Fig. 1 clearly shows two peaks with periods of 10.6 and 2.4 years (see Fig. 2). A very weak peak with a period of 4.7 years is also indicated. A performed significance test which is described in [28] gave significant results on the 95% level for the intervals

$$7.6 \leq 10.6 \leq 14.2 \text{ years and } 2.2 \leq 2.4 \leq 2.5 \text{ years.}$$

Fig. 2. Relative spectral density of the yearly rise of the lake level

White noise was assumed as the background spectrum, which seems reasonable from Fig. 2. The performed test is based on the assumption that the maximum entropy spectral density estimates are asymptotically distributed normal. The precise mathematical conditions for the validity of this assumption are given by Berk [7] in his theorem 6. An important condition is the flatness of the model spectrum. The *SNR* (8) can serve as an indicator of the flatness since it is the ratio of the arithmetic mean to the geometric mean of the model spectrum [26]. The *SNR* is close to unity in the presented case (see below) so we feel that the reality of the spectral peaks can be judged by this test. But it must be stressed that in general the performed test can give no more than vague hints; there is no consensus of opinion in literature about the distribution of the maximum entropy spectral density estimates and there is no one-to-one correspondence between spectral peak height and power [5, 24]. Recently Newton and Pagano [27] derived simultaneous confidence bands using Scheffé projections. We intend to incorporate these confidence bands into our algorithms.

Some information shall be added which in the opinion of the authors should be present in any published maximum entropy spectral estimation. Since the main drawback of this method is still the absence of an adequate significance test procedure this information can give hints about the reliability of any presented spectral density.

Applied algorithm: Burg algorithm [3, 4, 13]

Number of data points: 67

Upper limit *mmax* for searching the optimum filter order: 30

Optimum filter order as indicated by *FPE*, *CAT*, *OBD*: 6, 6, 0

Applied filter order: 6

Number of estimated spectral values: 509

Signal to noise ratio as defined in (8): 1.30975

The upper limit *mmax* might be too large but is still below the critical filter order [38]. The order selection 0 of *OBD* is probably caused by the small number of data points and the low signal to noise ratio.

The analysis shows that the level fluctuations of Lake Titicaca and the relative sunspot numbers have a spectral peak with a similar period. Further analysis conducted by Künzel [23] on the time series of the sunspot numbers for the same period (1914–1981) showed a prominent peak with a period of 10.1 years. Unfortunately it is not yet clear to which physical linkage a possible sunspot number-atmospheric circulation association can be attributed. But there are hints for a connection between sunspot numbers and tropopause height in the tropics. Stranz [34] and Cole [12] found out that the tropopause was higher in years of solar maximum compared to minimum. Since the tropopause is generally considered to mark the upper limit of convective activity of the lower atmosphere the idea is near at hand that

the compression of the troposphere could result in a higher precipitation activity. Investigations about the connection between tropopause height and precipitation amount on the Altiplano are in preparation.

The peak at 2.4 years [25] can be associated with the stratospheric quasi-biennial wind and temperature oscillations (QBO) in the tropics [14, 17]. Further investigations are planned to enlighten the connections between stratospheric wind and temperature oscillations and the precipitation regime within the region of the Altiplano. As Cole [12] has shown to be probable there also is a connection between the quasi-biennial oscillation and the variation of tropopause height.

The very weak peak at 4.7 could be a hint to the El Niño phenomenon. But it was pointed out earlier that a close relationship between the El Niño phenomenon and the level fluctuations of Lake Titicaca could not be observed as yet.

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