Indicators for Measuring Fiscal Sustainability
A Comparative Application of the OECD-Method and Generational Accounting

Ulrich Benz
Stefan Fetzer

118/04

INSTITUT FÜR FINANZWISSENSCHAFT
DER ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG IM BREISGAU
Indicators for Measuring Fiscal Sustainability
A Comparative Application of the OECD-Method and Generational Accounting

Ulrich Benz
Albert-Ludwigs-Universität Freiburg

and

Stefan Fetzer
Albert-Ludwigs-Universität Freiburg,

Original Version July 2004
Revised Version June 2005

Abstract
Based on an empirical application for Germany, we compare two methods measuring fiscal sustainability: the Generational Accounting approach and the OECD-Method. We show that both methods can be transferred into each other. Therefore the indicators of Generational Accounting can also be used for the OECD-Method (and vice versa) and consequently the set of sustainability indicators for both methods is enlarged. Furthermore we evaluate these indicators under the terms of theoretical deficiencies, tangibility and sensitivity. Finally we conclude that a combination of indicators stemming from both approaches can give a generally understandable description of fiscal sustainability and satisfy a strictly defined condition of sustainability at the same time.

JEL classification: E62, H62

* We are grateful to two anonymous referees for helpful comments and critiques.
Corresponding author. Forschungszentrum Generationenverträge, Albert-Ludwigs-Universität Freiburg, D-79085 Freiburg, stefan.fetzer@vwl.uni-freiburg.de.
1. Introduction

Traditional indicators of fiscal activity like public debt or annual public deficit fail to measure the sustainability of government’s policy as they only capture the short-term effects of current political decisions. For this reason the question of fiscal sustainability, which takes into account the long-term effects of the fiscal policy, became an important field in the economic literature.

Methods considering the long-term implications of today’s government’s decisions and their interplay with the upcoming demographic transition process have been developed at the beginning of the 1990s. The two most applied approaches so far are the method of Generational Accounting developed by Auerbach, Gokhale and Kotlikoff (1991, 1992, 1994) and the OECD-Method which is based on the fiscal sustainability concept proposed by Blanchard et al. (1990).

Both concepts are based on similar theoretical assumptions. In particular the common starting point is the intertemporal budget constraint of the government which is supposed to be fulfilled. Likewise the OECD-Method as well as Generational Accounting share similar theoretical and empirical shortcomings: Most importantly, general equilibrium effects are neglected in both concepts. Furthermore results from both methods heavily depend on data reliability and assumed parameter values. The latter is due to the fact that both concepts consider a long (OECD-Method) respectively infinite (Generational Accounting) time horizon. Another important problem of both methods is their strong dependence on business cycles which is due to the use of one particular base year. In this context one should note that the two concepts are no forecasting methods. Both approaches “only” try to evaluate the sustainability of a base year’s fiscal policy.

Despite these problems and because of the lack of other easily applicable methods, a wide range of country studies, e.g. for the European Union, have been conducted with both approaches. Furthermore the method of Generational Accounting became an official application in the USA between the years 1993 and 1995 and it is still used for preparing annual budgets in Norway.

---

1 See Ballasone and Franco (2000) for an overview.
2 Other approaches to examine fiscal sustainability are empirical studies based on fiscal data of the past. Recent studies in this direction are done by Afonso (2005) and Bohn (2005).
5 See CBO (1995). After 1995 Generational Accounting was taken out of official reports after having identified President Clinton’s health care reform proposal as being unsustainable.
Until now, Germany has failed to implement an official concept to measure fiscal sustain-
ability in government budgets. In 2001 several fiscal experts discussed the advantages and
disadvantages of both approaches and their appropriateness of being a regular part of public
statistics. The Advisory Council of the German Ministry of Finance (2001) and Kitterer
(2002) on the one side favor the OECD-Method. Their main criticism belonging on the Gener-
erational Accounting approach is the underlying infinite time horizon which they claim to be
inadequate in an official application. In contrast, Raffelhüschen (2002) points out that the
choice of a finite time horizon leads to arbitrary results of the OECD-Method. All in all these
experts hold forth about detailed differences and hence it seems to be that both concepts are
mutually exclusive.

Contrary to this discussion which is mainly based on theoretical considerations this paper
emphasizes the equivalence of both measuring concepts and compares them on the basis of an
empirical application: The sustainability of the current German fiscal policy. We choose an
approach which allows the application of indicators belonging to Generational Accounting to
the OECD-Method and vice versa. Consequently the set of sustainability indicators can be
expanded for both methods. In addition to the empirical comparison of the results derived
from both methods, we further evaluate the quality of these sustainability indicators.

Alternatively one could compare both methods and evaluate the quality of the indicators
algebraically. However, the results of the sustainability indicators highly depend on the inter-
action between the demographic projection, the base year’s governmental budget and the pa-
rameter values assumed for interest and growth development. A purely algebraical procedure
is not capable to catch the interaction effects taking place.

The remainder of this paper is organized as follows: Chapter 2 gives the definition of fiscal
sustainability used by both approaches. Furthermore, similarities and differences of both
methods in previous applications are pointed out. Then, Chapter 3 shows the results of several
sustainability indicators for the current German fiscal policy derived from both methods.
Based on these results Chapter 4 discusses and evaluates the quality of the indicators. Finally,
Chapter 5 summarizes our findings and concludes.

2. The Concept of Fiscal Sustainability

In their study Ballassone and Franco (2000) point out that no plain definition of fiscal sustain-
ability exists. Nevertheless there is one main aspect that is common to most fiscal sustainabil-

---

7 Even though Generational Accounting was used in the annual report of the German Council of Economic Ex-
erts (2003).
8 An algebraical comparison of both methods is given by Besendorfer (2004).
ity concepts which goes back to Domar (1944). The starting point is the government’s intertemporal budget constraint, which can be derived from the annual budget constraint (1) as follows:

\[ E_t + i \cdot D_{t-1} = R_t + (D_t - D_{t-1}) \]  

(1)

In each year, government expenditures, \( E_t \), and interest payments on public debt, \( i \cdot D_{t-1} \), have to be financed through revenues, \( R_t \), and the new public indebtedness, \( D_t - D_{t-1} \). Thus, public debt, \( D_t \), at any time \( t \) can be written as:

\[ D_t = E_t - R_t + (1 + i) \cdot D_{t-1} \]  

(2)

The accumulation of the public debt over several periods \( j = 1 \ldots t \) leads to:

\[ D_t = D_0 \cdot (1 + i)^t + \sum_{j=1}^{t} (E_j - R_j) \cdot (1 + i)^{t-j} \]  

(3)

Under the assumption that the real interest rate \( i \) is constant over time and equal to the interest rate on public debt, the present value of public debt in period \( t \) is equal to the initial public debt of period 0 plus the present value of all primary deficits, \( E_j - R_j \), for the \( t \) periods:

\[ \frac{D_t}{(1+i)^t} = D_0 + \sum_{j=1}^{t} \frac{(E_j - R_j)}{(1+i)^j}. \]  

(4)

In order to achieve a definition of fiscal sustainability, the following crucial condition has to be fulfilled: After an infinite time horizon \( t \) public debt is not allowed to be higher than its initial level, i.e. \( D_t \leq D_0 \). Thus, as \( t \) tends to infinity the discounted value of sustainable debt goes to zero, \( \lim_{t \to \infty} \frac{D_t}{(1+i)^t} = 0 \). Using this so called transversality condition in combination with equation (4), we obtain the sustainability condition used in most fiscal sustainability concepts:\n
\[ \sum_{j=1}^{\infty} \frac{(E_j - R_j)}{(1+i)^j} = -D_0 \]  

(5)

Thus, fiscal sustainability means that over an infinite time horizon the present value of all future primary deficits must equal the initial negative public debt. In other words, this implies

\[ \text{See the appendix for a detailed algebraical derivation and some notes on further conditions which have to be fulfilled in both concepts.} \]

\[ \text{In particular the OECD-Method expresses equation (5) in terms of ratios to GDP. See the appendix for a algebraical derivation of the equivalence of the two concepts despite this different expressions. With a somewhat different approach Alfonso (2005) gets the same result.} \]
an extinction of the current debt because its value tends to zero in the infinite horizon. Therefore, in present value terms all future expenditures have to be covered by all future revenues.

It should be noted that this definition is only true in the case of a positive so called “Aaron gap”, i.e. \( i - g > 0 \), which is valid for most developed countries\(^{11}\). If the annual growth rate \( g \) exceeded \( i \) a Ponzi game would be feasible and the question of sustainability would be superfluous because the government would be able to finance its interest payments and a part of the other expenditures by new debt issues without the need of generating primary surpluses. Put differently: The government could run permanent primary deficits without breaking the intertemporal budget constraint.

Despite this common definition of sustainability, the two methods differ in two essential points in their applications: the way of projecting payment flows and the time horizon.

The first difference is that usually *Generational Accounting* studies use a projection of expenditures and revenues at an individual level. Here, in a first step, age- and gender-specific profiles are rescaled to the initial public revenues and expenditures. In a second step these rescaled profiles are extrapolated into the future by using the constant annual real growth rate \( g \). For all future years, these extrapolated rescaled profiles are multiplied with the number of the respective age-group members, which are derived from a population projection. This third step leads to the future revenues and expenditures of the public sector.

The *OECD-Method* employs two different ways for projecting revenues and expenditures. The first one is used for all revenues and so called non-age-specific expenditures. They increase with the same rate as GDP, which means that an additional GDP-projection is needed. The second way of projecting is applied to all future age-specific expenditures. Their extrapolation procedure also varies between different studies as well as the assumption what kind of expenditures are defined as age-specific and non-age-specific respectively. As in Blanchard et al. (1990) and Franco and Munzi (1997) pension payments are assumed to grow in accordance with official projections from governments, projections of health or education expenditures are often done by the *Generational Accounting* procedure. In addition, the future age specific expenditures are sometimes assumed to grow with different growth rates, e.g. health expenditures grow with a higher rate than education expenditures.

The second essential difference between the *OECD-Method* and *Generational Accounting* is the underlying time horizon in applied studies. In *Generational Accounting* studies the time horizon is - consistent with equation (5) - infinite. In particular most studies compute the projections of public revenues and expenditures exactly for the next 306 years. After that point of

\(^{11}\) In the context of the Solow Model a positive “Aaron gap” implies a dynamic efficient growth path of the economy.
time the flow of payments is assumed to be constant. In contrast, *OECD-Method* studies assume a finite time horizon, mainly because this is supposed to find a broader acceptance in society. However, using a finite time horizon violates the sustainability condition of equation (5). Hence, the underlying sustainability condition for the *OECD-Method* procedure must rather be derived by solving equation (4) for the negative debt:

\[
\sum_{j=1}^{i} \left( \frac{E_j - R_j}{(1+i)^j} \right) - \frac{D_i}{(1+i)^j} = -D_0. \tag{4'}
\]

Now the sum of present values of all future primary deficits and the public debt at the end of the chosen time horizon must be equal to the initial negative public debt. In this context, the ratio of public debt to GDP at the end of the time horizon is often assumed to be the same as the initial ratio. However, the amount of public debt at the end of the time horizon can in fact have any conceivable value. Consequently, the so called sustainable debt (quota) at the end of the time horizon is determined normatively. But this has an impact on which generations have to pay for the debt, since all developments beyond the time horizon are not considered at all in the evaluation of the current fiscal stance. All in all it should be kept in mind that this procedure does not correspond to the sustainability definition in the strict sense of equation (5) because the transversality condition does not hold.

Apart from these two major differences, another point can be noted: Within the method of *Generational Accounting* it is also possible to add up revenues and expenditures for each generation. Then for each generation a so called generational account can be constructed by dividing all future net payments between the state and the members of one generation by the number of generation members in the base year. With these generational accounts intergenerational distribution effects can be shown as well and therefore examine the broader definition of sustainability propagated by the WCED (1987): "Sustainable development is development that meets the needs of present without compromising the ability of future generations to meet their own needs." The OECD-Method instead does not consider the aspect of intergenerational fairness and justice at all. Thus we neglect this point and concentrate our comparison between both methods on their question in common, namely, if current fiscal policy is sustainable or not.

The two essential differences between the two methods, the way of projecting payment flows and the time horizon, are shown in table 1. This classification is the basis for the sus-

---

12 In the original version of this concept Blanchard et al. (1990) proposed a time horizon of 40 years. Franco and Munzi (1997) examined fiscal sustainability after a time horizon of 32 years.

13 Some authors define the sustainable debt (quota) as the amount of debt (quota) which the government is able and willing to extinguish. Other sustainability definitions use a certain constant maximum debt quota like the 60 percent criterion in the European stability and growth agreement, cf. Blanchard (1984), IMF (1996), European Commission (1997) and Collignon and Mundschenk (1999).
tainability indicators, which will be introduced in the next chapter. So far, studies using a Generational Accounting approach mainly focus on indicators based on present value accounts with an infinite time horizon, whereas in OECD-Method studies the indicators are frequently denoted in terms of ratios to GDP over a certain time period. Using the two methods of projecting revenues and expenditures for each indicator, one can use indicators belonging to the OECD-Method for Generational Accounting and vice versa. By doing so, the fields I and II of table 1 can be completed, and the two methods can be compared using the same indicators. Thus, the set of indicators is enlarged and more importantly, the methods can be transferred into one another. Contrary to what the discussion between German fiscal experts – mentioned in the introduction – suggests, it becomes obvious that there is no real difference between the two methods. In fact, Generational Accounting and the OECD-Method are variations of a common sustainability definition.

[PLEASE INSERT TABLE 1 ABOUT HERE]

3. Sustainability Indicators – An Empirical Application for Germany

3.1. The data

The calculation of the sustainability indicators in this chapter is based on the Generational Accounting study of the German Council of Economic Experts (2003). The data covers, firstly, national accounts for revenues and expenditures of the public sector for the base year 2002, secondly, gender- and age-specific profiles mainly derived from the Sample Survey of Income and Expenditure, which is conducted by the Federal Statistical Office Germany (2001), and thirdly, the tenth coordinated population projection from the Federal Statistical Office Germany (2003a). To calculate future revenues and expenditures it is assumed in our projection that (supplementary to the procedure described in Chapter 2) people living in East Germany will reach the fiscal efficiency of their West German contemporaries during the next 40 years, and that medium- and long-term effects of currently decided reforms, especially those of the last pension and tax reforms, will actually be implemented. Finally, in the standard scenario the real interest rate \( i \) is chosen to be 3 percent and the real growth rate \( g \) is assumed to be 1.5 percent.

For the empirical realization of the OECD-Method our procedure follows the one used by Franco and Munzi (1997) in their study for the European Commission. Here the future development of all revenues and all non-age-specific expenditures of the public sector depends on the growth of GDP. All age-specific expenditures, which we defined to be all social benefits
from government or social insurance systems as well as all education expenditures are projected analogous to the *Generational Accounting* approach. Such a procedure may seem very simple compared to Franco and Munzi (1997), who among other things used an official government forecasting and different growth rates for projecting age specific expenditures. But since our objective is the comparison between the *Generational Accounting* approach and the *OECD-Method*, our procedure is more appropriate as both methods now only differ in extrapolating revenues and non-age-specific expenditures.

Essential for extrapolating revenues using the *OECD-Method* is the development of future GDPs. For this purpose we, again according to Franco and Munzi (1997), assume that the GDP per worker increases with the constant (labor productivity) growth rate $g$. Then future GDPs can be derived from a projection of future labor force. In order to obtain the latter, we supplement the population projection with age specific labor force participation rates, whereas they are assumed constant for the future\(^{14}\). In accordance to the *Generational Accounting* approach the East German workers’ fraction in one age-group will “catch up” to the West German level during the next 40 years.

The following sustainability indicators are each computed with the *Generational Accounting* projection method as well as with the one used by the *OECD-Method*. Furthermore we will distinguish between indicators based on an infinite time horizon (Chapter 3.2.) and indicators based on a finite time horizon (Chapter 3.3.).

### 3.2. Indicators with an infinite time horizon

#### 3.2.1. Sustainability gap

Under the assumption that the sustainability condition of equation (5) does not hold, the intertemporal public liabilities, $IPL_0$, can be written as:

$$IPL_0 = D_0 + \sum_{j=1}^{x} \left( \frac{E_j - R_j}{(1+i)^j} \right)$$

If the $IPL_0$ are greater than zero, the government neglects the intertemporal budget constraint and the sustainability condition does not hold (clearly $IPL_0$ equal zero corresponds to a sustainable fiscal policy). The $IPL_0$ comprise the present value of the entire public debt that occurred in the past and will arise in the future. The $IPL_0$ serve as a starting point for all indicators with an infinite time horizon.

---

\(^{14}\) Hence, we assume a production function with labor as the essential input factor. The labor participation rates stem from the Federal Statistical Office Germany (2003b).
The outcome of the $IPL_0$ in relation to the GDP of the base year is the so called *sustainability gap*, $SG$:

$$SG_0 = \frac{IPL_0}{Y_0} \quad (7)$$

The indicator $SG_0$ is easy to understand because it is very similar to the well-known debt to GDP ratio. But $SG_0$ not only refers to the debt that arose in the past, but also takes into account future liabilities. Thus the *sustainability gap* is often separated into two parts, namely the explicit and the implicit debt\(^{15}\). The explicit debt refers to the current debt and shows which part of future liabilities is based on the fiscal policy of the past. Note that in theory the present value of all future interest payments is equal to the current debt. The implicit debt on the other hand displays the present value of all future primary deficits and shows, which part of these liabilities are expected in the future due to the current fiscal policy.

Using the data described in section 3.1., the *sustainability gap* amounts to 331.3 percent of the base year GDP, if expenditures and revenues are projected with the *Generational Accounting* approach. Using the *OECD-Method* projection procedure, the outcome is a *sustainability gap* amounting to 477.7 percent of GDP. As a first result, we can conclude that the current fiscal policy is not sustainable and this result is independent of the chosen projection method. Using the *Generational Accounting* (*OECD-Method*) projection approach, the implicit debt amounts to 270.5 (416.9) percent of GDP and is 4.5 (7) times higher than the explicit debt which amounts 60.8 percent in 2002. The quantitatively higher value that results from the *OECD-Method* is due to the fact that revenues are extrapolated with the future growth rate of GDP, which is determined by the future development of the labor force. And because in ageing societies the labor force will decline faster than the population as a whole, the revenues related to GDP will decline faster than the revenues that are projected with the *Generational Accounting* framework. Within the latter, also pensioners have to pay e.g. taxes. On the other hand, the increasing age specific expenditures related to the increasing old age groups during upcoming decades is common to both methods because age specific expenditures are extrapolated in the same way.

\(^{15}\) See Jägers and Raffelhüschen (1999).
3.2.2. Annual consolidation

The following indicator is based on the idea that the \( IPL_0 \) can be completely offset by using a constant fraction \( \alpha \) of future GDPs for budgetary consolidation. Hence, \( \alpha \) can be computed as the ratio of the \( IPL_0 \) to the present value of all future GDPs\(^{16}\):

\[
\alpha = \frac{IPL_0}{\sum_{j=0}^{\infty} Y_j \cdot \frac{1}{(1+i)^j}}
\]

(8)

Thus, \( \alpha \) can be interpreted as a constant relative proportion of the populations’ productive power for each future year. This indicator can furthermore serve as a policy recommendation of how to establish a sustainable policy, without fixing the concrete government’s decisions which revenues should be increased or which expenditures should be reduced.

For the same reasons that were described in the discussion of the sustainability gap, \( \alpha \) is higher when applying the OECD-Method. Here, for each future year 8.88 percent of the GDP would have to be used to extinguish the intertemporal public liabilities, whereas this value amounts to 6.16 percent of future GDPs when extrapolating revenues and expenditures using the Generational Accounting approach\(^{17}\). In absolute numbers these results correspond to 187.2 and 129.86 Billion \( € \) in 2002. Both values are significantly higher than the 65 Billion \( € \) the German government spent for interest payments of outstanding debt in that year.

3.2.3. Adjustment of the tax quota

Both indicators, the sustainability gap as well as the annual consolidation, are not able to give precise recommendations how the government could deal with an unsustainable situation. One possibility to restore sustainability could be a proportional increase of all future taxes and contributions. Dividing \( IPL_0 \) by the present value of all future taxes and contributions \( (T) \), yields the tax quota’s increase, \( \theta \), that would be needed in order to achieve sustainability\(^{18}\):

\[
\theta = \frac{IPL_0}{\sum_{j=0}^{\infty} T_j \cdot \frac{1}{(1+i)^j}}
\]

(9)

By multiplying the base year’s tax quota with \((1 + \theta)\) we get the sustainable tax quota. We can thus state, that for eliminating the intertemporal public liabilities, all future taxes and con-

---

\(^{16}\) See Boll (1996) and Manzke (2002).

\(^{17}\) This value is close to the value of almost 6 percent the Bundesbank (2004) has calculated.

\(^{18}\) In an alternative interpretation \( \theta \) could reflect a proportional reduction of all transfers or \( \theta \) could be calculated so that only certain generations have to decompose the sustainability gap, see Raffelhüschen (1999) for details.
tributions have to be increased by 14.5 (22.5) percent when the aggregates are extrapolated using the *Generational Accounting (OECD-Method)* approach. This corresponds to a sustainable tax quota of 47.0 (50.3) percent of GDP instead of an actual tax quota of 41.1 percent in the year 2002. The quantitative difference between the two methods again stems from the different extrapolation of the revenues in this case, especially from the taxes and contributions in the denominator of equation (9). Yet, no matter which method is applied, this indicator shows the enormous dimension of restoring sustainability, since all taxes and contributions have to be increased by over 10 percent.

### 3.2.4. Delayed adjustment of the tax quota

Constructing the sustainable tax quota, we implicitly assume that the government is able to enforce an immediate adjustment of all taxes and contributions. But this assumption is not very realistic as it usually takes a few years while research results enter society and pass the political process. In addition, decisions in democracies are influenced by so called policy cycles, i.e. no unpopular laws are passed before important elections. Therefore, we suggest a new indicator \( \theta_x \), which refers not only to the immediate adjustment of the tax quota, but also gives the necessary increase of the tax quota, if the adjustment is delayed by \( x \) years. Like \( \theta \), the indicator \( \theta_x \) can be written as:\(^{19}\)

\[
\theta_x = \frac{IPL_0}{\sum_{j=x}^{\infty} T_j \cdot \frac{1}{(1+i)^j}}
\]

The indicator \( \theta_x \) can be interpreted as the costs of a delay in the political decision-making process or contrary, the advantage of quick actions. Figure 1 shows that the change of the tax quota to restore sustainability increases disproportional to the years of delay. Calculating revenues and expenditures with the *Generational Accounting (OECD-Method)* approach leads – as \( \theta \) illustrated – to an immediate increase of the tax quota of 14.45 (22.51) percent. A delay of 5 years causes a required increase of 15.77 (24.72), a delay of 10 years already demands an increase of 17.25 (27.23) percent and a delay of 20 years necessitates an increase of the tax quota of 20.83 (33.23) percent.

\[\text{[PLEASE INSERT FIGURE 1 ABOUT HERE]}\]

---

\(^{19}\) Instead of a delay in the tax quota adjustment also an indicator with a delay of the *annual consolidation* could have been computed.
3.2.5. Soft transition

Another way to illustrate a more realistic reaction of the government to establish a sustainable policy, than it is the case with an immediate increase of taxes and contributions, can be described by the *soft transition* indicator. Here it is assumed that year by year all age-specific transfers, $Tr$, are reduced by a constant percentage point $z$ of the original level until a sustainable situation is given\(^{20}\). Equation (10) shows that the present value of all reduced transfer payments needs to equal $IPL_0$:

$$\sum_{j=1}^{t} z \cdot j \cdot Tr_j \cdot \frac{1}{(1+i)^j} = IPL_0 \tag{10}$$

The resulting indicator is the time $t$ during the transition process is going on. Alternatively, we can denote the level of transfers after the transition as a further indicator, which is given as $(100 - t \cdot z)$ percent of the initial level.

Reducing all social benefits from the government or social security systems and all education expenditures by one percentage point per annum, yields a transition time of 31 years when aggregates are forecast with the *Generational Accounting* procedure. Hence, only by the year 2033 a sustainable situation is achieved. The transfer level until then will be reduced to 69 percent of the original one in this year. Still worse is the situation when aggregates are projected with the *OECD-Method*. In this case the initial level reduces to 48 percent and the transition process is finished only in the year 2054. Even though both methods assume the same extrapolation of the reduced transfers, the dates when the transition process is finished differ substantial between the two extrapolating approaches. The reason for this is the different development of future revenues, which differs due to the demographic change especially in the period 2030 to 2060 between both methods.

3.3. Indicators with a finite time horizon

3.3.1. Sustainable tax quota of Blanchard

One of the main indicators of the *OECD-Method* is the sustainable tax quota developed by Blanchard et al. (1990)\(^{21}\). This indicator is quite similar to the sustainable tax quota derived by equation (9). Strictly spoken $\theta$ is the necessary increase of taxes and contributions on an individual basis. Thus the sustainable tax quota calculated in section 3.2.3. is constant over time only under certain conditions, because it depends on the population development. In con-

---

\(^{20}\) A similar way to close the sustainability gap is to reduce the transfers each year by a certain percentage point of the respective level the year before. See Borgmann and Heidler (2003), who describe this indicator in an examination of the German pension system.

\(^{21}\) Alternatively the sustainable transfer rate can be computed. In an alternative way Seitz (2002) computes the sustainable expenditure growth rates given the forecasted revenues.
trast, the sustainable tax quota of Blanchard is defined as a constant quota for all years in the considered time horizon. In the case of projecting the revenues with the OECD-Method both indicators are equivalent, because only in this case taxes and contributions grow with the same rate as the GDP. Therefore, the sustainable tax gap of Blanchard can also be determined with the help of a constant adjustment parameter $\theta_{\text{Blanchard}}$ which – for an infinite time horizon – is equivalent to $\theta$. A modification of the $IPL_0$ for a finite time horizon can be derived from equation (4'). Hence, similar to equation (9), the adjustment parameter $\theta_{\text{Blanchard}}$ can be written as:

$$
\theta_{\text{Blanchard,}t} = \frac{D_0 - \frac{D_t}{(1+i)^t} + \sum_{j=1}^{t} \frac{(E_j - R_j)}{(1+i)^t}}{\sum_{j=0}^{t} \frac{1}{(1+i)^j}},
$$

(11)

where $D_t = D_0(1+g)^t$, so that the finite sustainability condition of Blanchard et al. (1990), a constant debt quota after the end of the time horizon, is fulfilled. With the help of $\theta_{\text{Blanchard,}t}$ the sustainable tax quota of Blanchard can be calculated as $(1 + \theta_{\text{Blanchard,}t}) \cdot \frac{T_t}{Y_t}$. The necessary increase of the tax quota, which Blanchard et al. (1990) define as the "tax gap", is given by $\theta_{\text{Blanchard,}t} \cdot \frac{T_t}{Y_t}$.

As shown above, under an infinite time horizon, the value of $\theta$ amounts to 22.51 (14.45) percent when aggregates are extrapolated using the Generational Accounting (OECD-Method) approach. The corresponding value for the tax quota is 47.0 (50.3) percent of GDP and the corresponding “tax gap” would amount to 5.9 (9.2) percent of GDP. Assuming a finite time horizon of 40 years as in the study of Blanchard et al. (1990), the necessary adjustment of the tax quota, $\theta_{\text{Blanchard,}40}$, to ensure the same debt quota as the initial one, is 8.64 (14.10) percent. The difference in these results leads to the speculation that the assumed time horizon has a significant influence on the level of necessary adjustments. Figure 2 shows the $\theta_{\text{Blanchard,}t}$ that results from a finite time horizon between 20 and 200 years.

[PLEASE INSERT FIGURE 2 ABOUT HERE]

---

22 The reason for this is that by multiplying a constant quota with a constant factor automatically yields a constant quota and vice versa. See also Besendorfer (2004).

23 Separating revenues $R_j$ into taxes $T_j$ and other revenues $(R_j - T_j)$ and then solving equation (4') for the taxes yields the original equation introduced by Blanchard et al. (1990). Supplementary all equations are written in terms of GDP ratios there.
As can be seen from figure 2, with an increasing time horizon $\theta_{\text{Blanchard},t}$ converges to the value of $\theta$, which corresponds to the infinite time horizon. This result is independent of the chosen extrapolating procedure. Figure 2 also shows the error of using the sustainability condition of equation (4') when fiscal sustainability is defined according to equation (5): In a finite time horizon of 40 years, $\theta_{\text{Blanchard},40}$ only accounts for 60 percent of the value of $\theta$, independent of the chosen sustainability approach. A time horizon of 32 years as is assumed in the study of Franco and Munzi (1997) can only explain about 50 percent of the value that results supposing the sustainability definition of equation (5) holds.  

3.3.2. Development of the debt quota

All of the indicators introduced so far only show the entire dimension of an unsustainable situation. In contrast to this, the emergence of this dimension over time can be illustrated by the development of the debt quota. Thus the development of the debt quota can determine the consequences of current fiscal policy for the next few years.

Putting equation (2) in relation to the GDP yields the debt quota of the corresponding year. Due to the fact that this indicator is well-known in society, namely as the so called Maastricht criterion of the European Commission (1992), the urgency to exert reforms can be shown in a very illustrative way. Figure 3 depicts the development of the debt quota for the next 50 years. As was the case with the other indicators, the development of the debt quota is favorable when revenues and expenditures are projected with the Generational Accounting approach. For both methods the debt quota begins to increase exponentially around the year 2020 due to highly increasing primary deficits and interest payments. Almost independent of the calculating method we get the following quantitative result: Within the next few years the debt quota will exceed the 60 percent of GDP positioned in the Maastricht criterion of the European Commission. In about 20 years, the debt quota passes the value of 100 percent of GDP and in 40 years it even exceeds 300 percent.

[PLEASE INSERT FIGURE 3 ABOUT HERE]

---

24 Based on algebraical considerations Raffelhüschen (2002) only gives 40 percent explanatory power to the OECD-Method with a time horizon of 32 years. This differing result confirms our sight that an algebraical procedure cannot catch the interaction of the underlying data in an adequate manner.
3.3.3. Development of the deficit quota

The second important Maastricht criterion is the *deficit quota* which has to be kept below the benchmark of 3 percent of GDP. The annual deficit is composed of interest payments on debt and the primary deficit. The development of the *deficit quota* for the next 50 years is displayed in figure 4. Under our assumptions the benchmark of 3 percent will already be broken in the year 2004. Furthermore, it is only until the year 2021 (2014) that the *deficit quota* remains under 5 percent of GDP when aggregates are extrapolated with the *Generational Accounting (OECD-Method)* approach. After that the *deficit quota* accelerates to 20 (30) percent around the year 2050, whereas this development is mainly due to the demographic shifting.

3.3.4. Ratio of primary deficits to revenues

This indicator shows how the implicit component of the *sustainability gap* accumulates over time. While the present value of all future tax payments on the initial debt corresponds to the explicit debt, the present value of all future primary deficits equals the implicit debt. By putting the primary deficits into relationship to the revenues in each future year, it can be displayed of how much the revenues of one year have to be increased, so that no primary deficit arises in the relevant year.\(^{25}\) The advantage of this indicator is its independence on the assumed interest and growth rates since primary deficits as well as revenues depend on the assumed parameters in the same way.

Figure 5 shows the development of the ratio of primary deficit to revenues for the next 50 years. As before, projecting aggregates with the *Generational Accounting* approach draws a somewhat improved picture of the current fiscal policy. Nevertheless, the trend of both curves is very similar: Starting from 0.96 percent of the revenues, the increase of the primary deficit until 2005 amounts to 3.5 (4.0) percent of the revenues using the aggregate projection of the *Generational Accounting (OECD-Method)*, whereas this is primarily based on the decline of revenues due to the last German tax reform. Between 2005 and about 2015 the *ratio of primary deficits to revenues* decreases. The reason for this is the consideration of the East German “catch up” process, which leads to higher revenues. After this and due to the starting

\(^{25}\) Alternatively the relation between primary deficit and expenditures can be computed to show by how much expenditures have to be reduced so that no primary deficit occurs.
demographic transition process, the annual primary deficits increase to over 15.0 (27.0) percent of the annual revenues around the year 2050. Hence, this indicator likewise shows the necessity of substantial reforms if a fiscal disaster is to be avoided.

4. What's the Quality of Good Indicators?

The previous chapter showed that both methods can be transferred into one another. The remaining difference is only due to the way of projecting aggregates. The Generational Accounting method extrapolates all revenues and expenditures on an individual basis. In contrast to this very uniform framework, the extrapolating of the OECD-Method has no standardized instructions, neither which expenditures should be defined as age-specific nor in which concrete way these should be projected. This degree of freedom might however lead to deviations in the results. Moreover, considering the current debate about generational contracts it seems arbitrary to project health expenditures or pension payments in a different way than pension or health contributions. Due to the age specific extrapolation those expenditures depend mainly on the demographic transition process, whereas the future development of the contributions depend to the future development of GDPs. The Generational Accounting approach on the other hand distributes expenditures for defense e.g. with the help of an uniform profile in its strict projection procedure, so that the growth of expenditures for defense depend on the population development. In our opinion it is appropriate to develop a more refined projection method dependent on the single aggregate. In the case of defense expenditures it seems more realistic having the growth rate for this type of expenditure independent of the population growth. All in all, both methods of projecion have their weaknesses. Hence, for the future it seems appropriate to establish an official uniform extrapolating procedure which is reasonable for all budget positions. However, it is essential to keep in mind that the sustainability indicators introduced in the last chapter can only be as good as the underlying projecting procedure. We are now going to evaluate these indicators according to three criteria, namely “theoretical deficiencies”, “sensitivity”, and “tangibility”.

The first criterion, “theoretical deficiencies”, arises from the second essential difference of both methods, the imputed time horizon. As mentioned before, a finite time horizon is assumed in the OECD-Method and thus sustainability in its strictly speaking sense does not hold. This caveat applies for all indicators underlying a finite time horizon. Moreover, from a theoretical standpoint all indicators based on the GDP projection are flawed, as they generate

---

26 In the case of Generational Accounting the question of how to project the single aggregates is a methodological one. Because this approach has evolved to illustrate the development of all governmental budget positions in the face of current demographic projections. But as mentioned above not all budget positions depend on demographic changes and therefore the projection method seems not adequate to us for all budget positions.

27 A hybrid projection approach for the German federal states was recently developed by Besendorfer (2004).
a higher uncertainty of the results according to the supplementary assumptions which are necessary for projecting the GDP.\(^{28}\)

The second criterion, “sensitivity”, arises from differing results when the parameters \(g\) and \(i\) are changed. As shown by Aaron (1966), a change in the results only depends on the deviation between growth and interest rate. In tables 2a and 2b the results for all indicators are derived from a low (\(g = 1.5\) percent, \(i = 2.5\) percent), a high (\(g = 2.0\) percent, \(i = 4.0\) percent) and a middle (\(g = 1.5\) percent, \(i = 3.0\) percent) growth-interest-difference (gid), also known as the Aaron factor. With the help of the variation of the results starting from the middle difference, the sensitivity of an indicator can be illustrated. This variation is displayed in table 2a and 2b in italicized and bold letters.

The indicator sustainability gap is very sensitive when the parameters \(g\) and \(i\) are changed. Its absolute values vary between 257.5 and 785.3 percent of GDP when the aggregate extrapolating is done according to the Generational Accounting method. When the projection is computed according to the OECD-Method the variation is between 367.8 and 1,161.0 percent of GDP. This corresponds to a difference between the low (high) and middle growth-interest-difference about 140.0 (22.0) percent. The fact that the sustainability gaps decrease when the Aaron factor is increased is due to declining future primary deficits in present value terms when the growth-interest-difference is raised.

The reaction of the indicator annual consolidation tends to the same direction, but with a lower sensitivity. The values fluctuate only by 3 (4) percent around the amount for the middle growth-interest-difference when aggregates are projected with the Generational Accounting (OECD-Method) approach. This outcome is due to the fact, that the \(IPL_0\) and the present value of future GDPs react in the same way to a change in the growth-interest-difference.

The sensitivity of the indicator adjustment of the tax quota is very similar to the sensitivity of the indicator annual consolidation. Here, the reactions on variations of growth and interest rates of the present value of future tax and contribution revenues is of the same kind as the reaction of future primary deficits. This robustness also holds for the delayed adjustment of the tax quota when short lags of 5 and 10 years are considered. But within this period the quality of the indicator changes, because \(\theta_{s=5}\) decreases, whereas \(\theta_{s=10}\) increases when the

\(^{28}\) In order to show this we assumed not only that GDP relates to the work force, which is assumed to be constant within an age group. We further presumed a second scenario, where the GDP grows with \(g\) and is thus independent of the demographic development. The quality of the results derived from both scenarios is the same, but quantitatively it highly varies.
Aaron Factor is increased. Presuming a delay of 25 years again leads to a higher sensitivity of this indicator. The indicator soft transition is also very robust and its reaction on changing growth and interest rates tens in the same direction as the sustainability gap.

To prove the sensitivity of the indicator sustainable tax quota of Blanchard we use the $\theta_{\text{Blanchard,40}}$ which results from a time horizon of 40 years, as Blanchard et al. (1990) propose. The results for this indicator range around the middle growth-interest-difference and thus, it is akin to $\theta$. Although $\theta$ decreases, $\theta_{\text{Blanchard,40}}$ increases when the Aaron factor is increased. At a first glance, this seems to be surprising, but the reason for this converse reaction of the two indicators belongs to the assumed time horizon. Considering equation (9) and (11) one can see that the nominator as well as the denominator decline when the interest rate is raised, so that it cannot be stated a priori whether the indicator’s reaction goes upwards or downwards.

The sensitivity of the indicators debt quota and deficit quota is examined for the years 2027 and 2052. The indicators’ deviations from the middle growth-interest difference are on a relative high level which are even higher when the later point in time is considered. In contrast to the sustainability gap the indicators debt quota and deficit quota raise when the Aaron factor is increased. The reason for this is, that the future GDPs of the years 2027 and 2052 – to which the debt as well as the deficit is related to – are increasing with $g$, whereas the sustainability gap is related to the base year’s GDP, which is not influenced by changes in $g$. The by far most robust indicator is the ratio of primary deficits to revenues. It is totally independent of the assumed growth-interest-difference, due to the same reaction of primary deficits and revenues on changing parameters.

The third evaluation criterion, “tangibility”, refers to the practicability for policy and society. The indicators debt quota and deficit quota meet this criterion best since they are well-known as the Maastricht criteria. Both indicators illustrate the development of the fiscal imbalance over a certain period of time. This development is also shown by the indicator ratio of primary deficits to revenues for the annual evolving part of the debt. But in contrast to the debt quota and deficit quota, the ratio of primary deficits to revenues is not positioned in the public awareness.

With respect to the criterion “tangibility” all indicators underlying an infinite time horizon have the problem that calculations on the basis of infinity are neither imaginable nor comprehensible for the broad public. Within the group of indicators based on an infinite time horizon, the sustainability gap however corresponds best to the “tangibility” criterion for two reasons: First, the $IPL_0$ are often labeled as the true debt and therefore the sustainability gap is nothing else than a superior version of the well-known base year’s debt quota. Second, the sustainability gap is often used to make an illustrative and comprehensible comparison of
long term implications of different reforms or reform proposals. Here, the other indicators of this group are less illustrative due to their lower quantitative values. Besides that, the indicators annual consolidation and soft transition are appropriate to show the necessary adjustment of fiscal policy in a very simple way. The same holds for the indicators $\theta$, $\theta_\epsilon$ and $\theta_{\text{Blanchard},t}$, since they show the required adjustment for the average citizen on a microeconomic level. Moreover, it is easy to transform these indicators into the well-known tax quota on a macroeconomic level.

To summarize, table 3 presents an evaluation of the indicators in respect of the three criteria “theoretical deficiencies”, “sensitivity” and “tangibility”. In principle there exists a trade-off between the criteria “tangibility” and “sensitivity”. This is true especially for the indicators debt quota, deficit quota and sustainability gap. But for the latter, the bad assessment with respect to the criterion “sensitivity” only holds when the sustainability gap is used in absolute terms. Applying this indicator for comparisons of reforms, one can show the reform induced change of the sustainability gap in percent. This percentage change reacts very robust to a change in the parameters as shown by the German Council of Economic Experts (2003). The deduction in evaluating the annual consolidation indicator via the criterion “theoretical deficiencies” is due to the fact that this indicator is based on the GDP projection for which supplementary assumptions are needed. The indicators $\theta$, $\theta_\epsilon$ and soft transition meet all three criteria in a relatively good manner. Despite the bad performance of the indicators debt quota and deficit quota with respect to the criteria “theoretical deficiencies” and “sensitivity”, these indicators should be applied in future studies due to their good assessment with respect to the criterion “tangibility”. As a result of its absolute robustness we propose the use of the indicator ratio of primary deficits to revenues, which can furthermore give an illustrative and helpful insight into fiscal consequences resulting from the demographic change. Hence, the only indicator we recommend to reject for future fiscal sustainability approaches is $\theta_{\text{Blanchard},t}$, as the underlying definition of fiscal sustainability is not the strict one of equation (5) and, secondly $\theta_{\text{Blanchard},t}$ is redundant to $\theta$.

5. Conclusions

The results of this paper show the need for a comprehensive reform of the German fiscal policy, if sustainability is a prior objective as it is often propagandized by policy makers. This result is independent of the underlying concept of measurement and the sustainability indica-
tors chosen. In this context, it could be shown that Generational Accounting and the OECD-Method are not contrary sustainability concepts. They even can be transformed into each other. Both concepts are based on the same theoretical background and differ only in their empirical applications. Due to the fact that indicators for both methods can be based on any time horizon, the first difference between Generational Accounting and OECD-Method, namely the (often discussed) assumed time horizon, does not hold any longer. The second difference is related to the way revenues and expenditures are calculated. In this respect the Generational Accounting procedure seems to be more uniform than the OECD-Method’s extrapolation and thus less vulnerable to (politically motivated) manipulation. On the other hand the microeconomic foundation for the calculation within the Generational Accounting framework for some budget positions is more than questionable. Hence, an experts’ discussion focusing on the extrapolating procedure for each budget position in order to develop a (hybrid) uniform concept for measuring sustainability officially, seems to be more fruitful than discussing which of the both methods is the better one.

Moreover we evaluated sustainability indicators with the criteria “theoretical deficiencies”, “sensitivity” and “tangibility”. As a central result of this evaluation it can be stated that the “perfect” sustainability indicator does not yet exist, since all indicators considered have advantages as well as disadvantages. Therefore, for future sustainability studies it seems appropriate to use a certain set of indicators. Thus, by using indicators both on basis of a finite and on basis of an infinite time horizon, the sustainability problem becomes comprehensible to the broad public and at the same time the sustainability concept is defined in its strict sense.

All in all by transferring both methods into one another we try to help bringing the fruitless quarrel about the superiority of one concept over the other to an end. Beside the establishment of a uniform official projecting procedure for the several budget positions further research efforts should be put into questions like the sensitivity of the results to business cycles or the influence of uncertainty about demographic, economic and fiscal parameters.
References


Afonso, A. (2005), Fiscal Sustainability: The Unpleasant European Case, Finanzarchiv 61, 19-44.


IMF – International Monetary Fund (1996), World Economic Outlook No. 5/96.


Raffelhüschen, B. (2002), Ein Plädoyer für ein flexibles Instrument zur Analyse nachhaltiger Finanzpolitik, Wirtschaftsdienst 82/2, 73-76.

Seitz, H. (2002), Sustainability of public finances at the state level: Indicators and empirical evidence for the German Länder, European University of Frankfurt/Oder.


### Table 1
Classification of the Generational Accounting approach and the OECD-Method

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>Projection</th>
<th>Individual Level</th>
<th>Individual Level and Aggregate Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite</td>
<td>Generational Accounting</td>
<td>I</td>
<td></td>
</tr>
<tr>
<td>Finite</td>
<td>II</td>
<td>OECD-Method</td>
<td></td>
</tr>
</tbody>
</table>
Table 2a
Sensitivity analysis of sustainability indicators with an infinite time horizon

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Aggregate projection with Generational Accounting</th>
<th>Aggregate projection with OECD-Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low gid</td>
<td>middle gid</td>
</tr>
<tr>
<td><strong>sustainability gap</strong> (% of base year GDP)</td>
<td>785.3</td>
<td>331.3</td>
</tr>
<tr>
<td></td>
<td>137.04%</td>
<td>-22.28%</td>
</tr>
<tr>
<td><strong>annual consolidation</strong> (% of present value of all future GDPs)</td>
<td>6.34</td>
<td>6.16</td>
</tr>
<tr>
<td></td>
<td>2.92%</td>
<td>-3.25%</td>
</tr>
<tr>
<td><strong>θ</strong> (adjustment of tax quota in %)</td>
<td>14.8</td>
<td>14.5</td>
</tr>
<tr>
<td></td>
<td>2.07%</td>
<td>-2.76%</td>
</tr>
<tr>
<td><strong>θ_{x=5}</strong> (adjustment of tax quota in %)</td>
<td>15.8</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>0.00%</td>
<td>-0.63%</td>
</tr>
<tr>
<td><strong>θ_{x=10}</strong> (adjustment of tax quota in %)</td>
<td>16.8</td>
<td>17.3</td>
</tr>
<tr>
<td></td>
<td>-2.89%</td>
<td>1.73%</td>
</tr>
<tr>
<td><strong>θ_{x=25}</strong> (adjustment of tax quota in %)</td>
<td>20.7</td>
<td>22.9</td>
</tr>
<tr>
<td></td>
<td>-9.61%</td>
<td>9.61%</td>
</tr>
<tr>
<td><strong>soft transition</strong> (level in % after transition)</td>
<td>71</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>2.90%</td>
<td>-2.90%</td>
</tr>
</tbody>
</table>

Table 2b
Sensitivity analysis of sustainability indicators with a finite time horizon

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Aggregate projection with Generational Accounting</th>
<th>Aggregate projection with OECD-Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low gid</td>
<td>middle gid</td>
</tr>
<tr>
<td><strong>θ_{Blanchard,40}</strong> (adjustment of tax quota in %)</td>
<td>8.2</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>-4.65%</td>
<td>5.81%</td>
</tr>
<tr>
<td><strong>debt quota in 2027</strong> (% of 2027’s GDP)</td>
<td>126.9</td>
<td>131.8</td>
</tr>
<tr>
<td></td>
<td>-3.72%</td>
<td>13.81%</td>
</tr>
<tr>
<td><strong>debt quota in 2052</strong> (% of 2052’s GDP)</td>
<td>384.7</td>
<td>442.3</td>
</tr>
<tr>
<td></td>
<td>-13.02%</td>
<td>65.05%</td>
</tr>
<tr>
<td><strong>deficit quota in 2027</strong> (% of 2027’s GDP)</td>
<td>6.9</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>-12.66%</td>
<td>24.05%</td>
</tr>
<tr>
<td><strong>deficit quota in 2052</strong> (% of 2052’s GDP)</td>
<td>15.4</td>
<td>21.7</td>
</tr>
<tr>
<td></td>
<td>-29.03%</td>
<td>31.34%</td>
</tr>
<tr>
<td><strong>primary deficit in 2027</strong> (% of revenues 2027)</td>
<td>7.8</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
Table 3
Evaluation of the sustainability indicators

<table>
<thead>
<tr>
<th>indicator</th>
<th>theoretical deficiencies</th>
<th>tangibility</th>
<th>sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>sustainability gap</td>
<td>+</td>
<td>++</td>
<td>- -</td>
</tr>
<tr>
<td>annual consolidation</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\theta$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>soft transition</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\theta_{Blanchard,t}$</td>
<td>- -</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>debt quota</td>
<td>-</td>
<td>+++</td>
<td>-</td>
</tr>
<tr>
<td>deficit quota</td>
<td>-</td>
<td>+++</td>
<td>-</td>
</tr>
<tr>
<td>ratio of primary deficits to revenues</td>
<td>+</td>
<td>-</td>
<td>++</td>
</tr>
</tbody>
</table>
Figure 1
Delayed adjustment of the tax quota
Figure 2
Necessary adjustment of the tax quota regarding different finite time horizons
Figure 3
Development of the debt quota
Figure 4
Development of the deficit quota
Figure 5

Development of the ratio of primary deficits to revenues

[Graph showing the development of the ratio of primary deficits to revenues over the years 2000 to 2050, with two lines representing Generational Accounting and OECD Method.]
Appendix

The equivalence of the sustainability criterion according to Blanchard et al. (1990) and the Generational Accounting approach:

The well-known sustainability criterion by Blanchard et al. (1990) is given by:\(^{29}\)

\[
-d_0 = \sum_{j=1}^{\infty} pd_j \frac{(1+g)^j}{(1+i)^j}
\]  
(A.1)

Where \( d_0 \) is the base year’s debt quota and \( pd_j \) indicates the ratio of primary deficits to GDP in year \( j \). Equation (A.1) results from the development of the debt quota in year \( t \) in present value terms

\[
d_t \frac{(1+g)^t}{(1+i)^t} = \sum_{j=1}^{t} pd_j \frac{(1+g)^j}{(1+i)^j} + d_0
\]

and the following transversality condition

\[
\lim_{t \to \infty} \left( d_t \frac{(1+g)^t}{(1+i)^t} \right) = 0
\]

(A.3)

The development of the debt can be described with ex post average growth rate \( \gamma \):

\[
D_t = D_0 \cdot (1 + \gamma)^t
\]

(A.4a)

Defining \( g \) as the ex post average growth rate of GDP yields:

\[
Y_t = Y_0 \cdot (1 + g)^t
\]

(A.4b)

Combining equation (A.4a) and (A.4b) the development of the debt quota can be written as:

\[
d_t = \frac{D_t}{Y_t} = \frac{D_0}{Y_0} \cdot \frac{(1 + \gamma)^t}{(1 + g)^t}
\]

(A.5)

Combining equation (A.5) and (A.3) yields:

\[
\lim_{t \to \infty} \left( \frac{D_0}{Y_0} \cdot \frac{(1 + \gamma)^t}{(1 + g)^t} \cdot \frac{(1 + g)^t}{(1 + i)^t} \right) = 0
\]

(A.6)

So it’s easy to see that the transversality condition used by Blanchard (1990) only holds if the ex post average growth rate of the debt \( \gamma \) is smaller than the real interest rate \( i \).

---

\(^{29}\) We use a discret version because of a better comparability with the Generational Accounting method. See Blanchard et al. (1990), p.10-12 for the whole mathematical derivation.
The same consideration in the *Generational Accounting* approach gets the following sustainability criterion (A.7) and transversality condition (A.8), which have to be fulfilled:

\[
\sum_{j=1}^{\infty} \left( \frac{pD_j}{(1+i)^j} \right) = -D_0 \quad (A.7)
\]

\[
\lim_{t \to \infty} \left( \frac{D_t}{(1+i)^t} \right) = 0 \quad (A.8)
\]

Using equation (A.4a) to describe \( D_t \) gets:

\[
\lim_{t \to \infty} \left( D_0 \cdot \frac{(1+\gamma)^t}{(1+i)^t} \right) = 0 \quad (A.9)
\]

As equation (A.6) also equation (A.9) is only true if the condition \( \gamma < i \) holds and thus, under an infinite time horizon it does not matter if the sustainability condition is defined in quotas or in absolute terms.
Seit 2000 erschienene Beiträge

82/00 Jochen Michaelis/Michael Pflüger
The Impact of Tax Reforms on Unemployment in a SMOPEC
erschienen in: Journal of Economics (Zeitschrift für Nationalökonomie), Vol. 72, No. 2, S. 175-201

83/00 Harald Nitsch
Disintermediation of Payment Streams

84/00 Harald Nitsch
Digital Cash as a Medium of Exchange: A Comment on the Application of the Whitesell-Model

85/00 Harald Nitsch
Efficient Design of Wholesale Payment Systems: The Case of TARGET

86/00 Christian Keuschnigg/Mirela Keuschnigg/Reinhard Koman/Erik Lüth/Bernd Raffelhüschen
Public Debt and Generational Balance in Austria

87/00 Daniel Besendorfer/Holger Bonin/Bernd Raffelhüschen
Reformbedarf der sozialen Alterssicherung bei alternativen demographischen Prognosen

88/00 Erik Lüth
The Bequest Wave and its Taxation

89/00 Hans-Georg Petersen/Bernd Raffelhüschen
Die gesetzliche und freiwillige Altersvorsorge als Element eines konsumorientierten Steuer- und Sozialsystems

90/00 Patrick A. Muhl
Der walrasianische Auktionator - wer ist das eigentlich?

91/00 Michael Pflüger
Ecological Dumping Under Monopolistic Competition
erschienen in: Scandinavian Journal of Economics, 103(4), S. 689-706

92/01 Christoph Borgmann/Pascal Krimmer/Bernd Raffelhüschen
Rentenreformen 1998 - 2001: Eine (vorläufige) Bestandsaufnahme
erschienen in: Perspektiven der Wirtschaftspolitik 2001, 2(3), S. 319-334

93/01 Christian Keuschnigg/Mirela Keuschnigg/Reinhard Koman/Erik Lüth/Bernd Raffelhüschen
Intergenerative Inzidenz der österreichischen Finanzpolitik

94/01 Daniel Besendorfer/A. Katharina Greulich
Company Pensions and Taxation

95/01 Bernd Raffelhüschen
Generational Accounting - Quo Vadis?

96/01 Bernd Raffelhüschen
Soziale Grundsicherung in der Zukunft: Eine Blaupause
erschienen in: B. Genser (Hrsg.), Finanzpolitik und Arbeitsmärkte, Schriften des Vereins für
97/01 Christoph Borgmann
Assessing Social Security: Some Useful Results

98/01 Karen Feist/Pascal Krimmer/Bernd Raffelhüschen
Intergenerative Effekte einer lebenszyklusorientierten Einkommensteuerreform: Die Einfachsteuer des Heidelberger Steuerkreises
er schienen in: Manfred Rose (Hrsg.): Reform der Einkommensbesteuerung in Deutschland, Schriften des Betriebs-Beraters Band 122, Heidelberg 2002, S. 122-145

99/01 Stefan Fetzer/Stefan Moog/Bernd Raffelhüschen
Zur Nachhaltigkeit der Generationenverträge: Eine Diagnose der Kranken- und Pflegeversicherung

100/02 Christoph Borgmann
Labor income risk, demographic risk, and the design of (wage-indexed) social security

101/02 Philip M.V. Hallensleben
Monetäre Transmission in Europa und Folgen für die Geldpolitik der Europäischen Zentralbank

102/02 Josef Honerkamp/Stefan Moog/Bernd Raffelhüschen
Earlier or Later in CGE-Models: The Case of a Tax Reform Proposal

103/02 Stefan Fetzer/Bernd Raffelhüschen
Zur Wiederbelebung des Generationenvertrags in der gesetzlichen Krankenversicherung: Die Freiburger Agenda
erscheint demnächst in: Perspektiven der Wirtschaftspolitik

104/03 Oliver Ehrentraut/Stefan Fetzer
Wiedervereinigung, Aufholprozess Ost und Nachhaltigkeit
er schienen in: Wirtschaftsdienst, Heft 4, 2003, S. 260-264

105/03 Pascal Krimmer/Bernd Raffelhüschen
Intergenerative Umverteilung und Wachstumsimpulse der Steuerreformen 1999 bis 2005 - Die Perspektive der Generationenbilanz

106/03 Stefan Fetzer/Stefan Moog/Bernd Raffelhüschen
Die Nachhaltigkeit der gesetzlichen Kranken- und Pflegeversicherung: Diagnose und Therapie
er schienen in: Manfred Albring/Eberhard Wille (Hrsg.): Die GKV zwischen Ausgabendynamik, Einnahmenschwäche und Koordinierungsproblemen, Frankfurt 2003, S. 85-114

107/03 Christoph Borgmann/Matthias Heidler
Demographics and Volatile Social Security Wealth: Political Risks of Benefit Rule Changes in Germany
er schienen in: CESifo Working Paper No. 1021

108/03 Stefan Fetzer/Dirk Mevis/Bernd Raffelhüschen
Zur Zukunftsfähigkeit des Gesundheitswesens. Eine Nachhaltigkeitsstudie zur marktorientierten Reform des deutschen Gesundheitssystems

109/03 Oliver Ehrentraut/Bernd Raffelhüschen
Die Rentenversicherung unter Reformdruck – Ein Drama in drei Akten

110/03 Jasmin Häcker/Bernd Raffelhüschen
Denn sie wussten was sie taten: Zur Reform der Sozialen Pflegeversicherung
erschienen in: Vierteljahrshefte zur Wirtschaftsforschung 73/1 (2004), S. 158-174

111/03 Harald Nitsch
One Size Fits – Whom? Taylorzinsen im Euroraum

112/03 Harald Nitsch
Aggregationsprobleme von Investitionsfunktionen im Immobilienbereich

113/04 Bernd Raffelhüschen/Jörg Schoder
Wohneigentumsförderung unter neuen Vorzeichen: Skizze einer zukunftsorientierten Reform

114/04 Stefan Fetzer/Christian Hagist
GMG, Kopfpauschalen und Bürgerversicherungen: Der aktuelle Reformstand und seine intergenerativen Verteilungswirkungen

115/04 Christian Hagist/Bernd Raffelhüschen
Friedens– versus Ausscheidegrenze in der Krankenversicherung: Ein kriegerischer Beitrag für mehr Nachhaltigkeit
erscheint demnächst in: Zeitschrift für die gesamte Versicherungswissenschaft

116/04 Sandra Haasis
Interbankenverrechnung – eine Bedrohung für die Europäische Geldpolitik? Nachfrage bei der Deutschen Kreditwirtschaft

117/04 Ulrich Benz/Stefan Fetzer
Was sind gute Nachhaltigkeitsindikatoren? OECD-Methode und Generationenbilanzierung im empirischen Vergleich

118/04 Ulrich Benz/Stefan Fetzer
Indicators for Measuring Fiscal Sustainability – A Comparative Application of the OECD-Method and Generational Accounting
Englische Version von Diskussionsbeitrag 117/04

ISSN 0943-8408